

MODULATIONAL INSTABILITY IN BOSE-EINSTEIN CONDENSATE IN OPTICAL SUPERLATTICE

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ABSTRACT

Steady state distribution of the probability amplitudes and the site population in the one dimensional optical superlattice was found. It was shown that this solution of the equations which describe the dynamics of the Bose-Einstein condensate in superlattice is unstable at the sufficiently high density of the bosons. The expression for increment of the modulational instability was found on base of the linear stability analysis. Numerical simulation demonstrates the evolution of the steady state distributions of bosons into the space array of the solitary peaks before the chaotic regime generation.

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1 Introduction

Interference of the several plane waves of monochromatic radiation can form a diffraction pattern in which the electric field strength periodically varies in space. Resulting periodic system of the microscopic potentials is designated as optical lattice [1, 2, 3]. Bose-Einstein condensates (BECs) trapped in optical lattice have been studied in sufficient detail [4, 5, 6]. The dynamics of atoms in an optical lattice can be described by two methods. The first method is based on the nonlinear Schrödinger equation with a periodic potential [7, 8]. In papers devoted to investigation of the Bose-Einstein condensation, this equation is often called the Gross-Pitaevskii equation [9, 10, 11]. The second method answering the tight-binding approximation is based on the Hubbard model, where the operators of creation and annihilation of fermions are replaced by the operators satisfying the commutation relations for bosons. The resulting model is often called the Bose-Hubbard model [12, 13, 14, 15].

Recently the optical lattices with two sorts of microscopic potentials in a unit cell have attracted attention [16, 17, 18, 19, 20]. In analogy with theory of solid state crystals, this periodic system of the microscopic potentials can be named optical superlattice. Due to the difference of the energy levels of microscopic potentials quantum tunneling between the nearest-neighbor sites is absent. Also, at low temperatures thermoactivated transport of an atom from one site of the micropotential to the other site is absent. However, the photo-induced transport of atoms along the optical superlattice is possible under the condition for the Raman resonance [12, 21, 22]. A similar process is known in nonlinear optics as a coherent population transfer and in solid state physics of low dimensional systems as coherent transfer of electrons or excitons in a system of coupled quantum dots.

Under CW electromagnetic radiation the different nonlinear excitations can propagate in the optical superlattice. It is important to emphasize that parameters of these excitations can be controlled by additional radiation, which defines hopping rates between the adjacent sites.

Frequently the steady state wave motion takes place in various physical systems. Due to the interplay between dispersive and nonlinearity effects a weak perturbation of the steady state wave may induce the exponential growth of the perturbation. That phenomenon is the modulational instability of a steady state wave. Some times modulational instability results in train of the soliton-like waves. But it does not always happen.

Modulational instability in BECs in the case of the ordinary optical lattices was investigated in [23, 24, 25]. It was shown that modulational instability is basic mechanism by which solitons are created in BEC. The kind of solitons (i.e., bright or dark one) depends on sign of the scattering lengths.

In this paper we consider the dynamics of the site populations for optical superlattice with two sorts of microscopic potentials in a unit cell. The generated Bose-Hubbard model describing this system in the tight-binding approximation was used in [21, 22] to write the system of equations of motion for probability amplitudes of the site population. These equations are employed as the basis for present investigation. We found the stationary distribution of the probability amplitudes. It should be pointed out that this distribution is not homogeneous one, as opposed to the case of ordinary optical lattices. The main result is an analytic expression for the modulational instability increment, which depends on the site population and the wave number of harmonic weak perturbations. This expression was obtained on base of the linear stability analysis. The numerical simulation shows that instability leads to strongly non-regular pattern.

2 Model and basic equations

Let us consider the one-dimensional optical superlattice with two kinds of sites in the tight-binding approximation [12, 13]. The microscopic potentials of one depth correspond to sites with an even number, and microscopic potentials of other depth correspond to sites with an odd number. Sites labeled by even numbers contain bosons in the ground state $|g_a\rangle$ with the energy ε_a . Sites labeled by odd numbers contain bosons in the ground state $|g_b\rangle$ with the

energy $\varepsilon_b > \varepsilon_a$. Let the temperature of the system be such that the higher levels of the microscopic potentials are not populated. Since the energies of the ground states of neighboring sites differ, the process of direct tunneling of an atom from one site to another one can be excluded from the consideration. Let us assume that biharmonic radiation (ω_1 and ω_2 are the frequencies of the carrier waves) acts on the atoms and the condition for the Raman resonance ($\varepsilon_b - \varepsilon_a \approx \hbar(\omega_1 - \omega_2)$) is fulfilled. In this case, after absorption of the first (second) photon, the atom will go from the deep (shallow) microscopic potential to the state of the continuous spectrum, and, after emission of the second (first) photon, it will return to the state having the energy ε_b (ε_a) and thus will be brought into a shallow (deep) microscopic potential. Thus, although tunneling or thermoactivated transport of atoms along the optical superlattice is absent, their photo-induced transport is possible.

In classical limit the system of equations describing the probability amplitudes of the populations for sites $a_{2j} = \langle \hat{a}_{2j} \rangle$ and $b_{2j+1} = \langle \hat{b}_{2j+1} \rangle$ takes the following form [21, 22]:

$$i\hbar \frac{\partial}{\partial t} a_{2j} = -J_0^* e^{i\Delta\omega t} + (b_{2j-1} + b_{2j+1}) + \varepsilon_{a,2j} a_{2j} + U_{aa} |a_{2j}|^2 a_{2j} + U_{ab} (|b_{2j-1}|^2 + |b_{2j+1}|^2) a_{2j}, \quad (1)$$

$$i\hbar \frac{\partial}{\partial t} b_{2j+1} = -J_0 e^{-i\Delta\omega t} + (a_{2j} + a_{2j+2}) + \varepsilon_{b,2j+1} b_{2j+1} + U_{bb} |b_{2j+1}|^2 b_{2j+1} + U_{ab} (|a_{2j}|^2 + |a_{2j+2}|^2) b_{2j+1}, \quad (2)$$

where $\Delta\omega = (\omega_1 - \omega_2)$, the parameters U_{aa}, U_{bb} define interaction between atoms induced by on-site atomic collisions and interaction between atoms of neighbor sites is defined by U_{ab} .

The first term in these equations takes into account the nearest-neighbor hopping induced by the stimulated Raman scattering. We assume that inhomogeneous broadening is absent, i.e., $\varepsilon_{a,2j} = \varepsilon_a$ and $\varepsilon_{b,2j+1} = \varepsilon_b$. If we introduce the control electromagnetic field amplitudes $\mathcal{E}_{1,2}$ then the nearest-neighbor hopping term read as $J_0 = \mu_{12} \mathcal{E}_1 \mathcal{E}_2^*$, where μ_{12} is the matrix element of the Raman transition. If one introduce $J_0 = |J_0| \exp(i\vartheta)$, and assume that

control electromagnetic fields have a constant phase, then ϑ can be included into complex value of the probability amplitudes b_{2j+1} . Thus we can substitute $b_{2j+1} \rightarrow b_{2j+1} \exp(i\vartheta) = \tilde{b}_{2j+1}$, after that suppose the parameter J_0 as real value. If the interaction between atoms of neighbor sites is neglected then the system of resulting equations takes the following form

$$\begin{aligned} i\partial\tilde{a}_{2j}/\partial\tau &= -\left(\tilde{b}_{2j-1} + \tilde{b}_{2j+1}\right) + \beta_a|\tilde{a}_{2j}|^2\tilde{a}_{2j}, \\ i\partial\tilde{b}_{2j+1}/\partial\tau &= -(\tilde{a}_{2j} + \tilde{a}_{2j+2}) + \delta\tilde{b}_{2j+1} + \beta_b|\tilde{b}_{2j+1}|^2\tilde{b}_{2j+1}, \end{aligned} \quad (3)$$

where $\Delta\varepsilon = (\varepsilon_b - \varepsilon_a) - \hbar\Delta\omega$, $\beta_a = U_{aa}/|J_0|$, $\beta_b = U_{bb}/|J_0|$, $\delta = \Delta\varepsilon/|J_0|$. We use the normalized time variable $\tau = t|J_0|/\hbar$. For the sake of simplicity we will assume that the exact resonance condition is hold, i.e., $\delta = 0$.

3 Stationary solution

It should remark that the atomic transport between neighbor sites in superlattice is absent if the phases of amplitudes a_{2j} and a_{2j+2} as well as b_{2j-1} and b_{2j+1} will be opposite one.

Fig. 1 represents schematically the probability amplitudes configurations in superlattice. Thin line arrows correspond to even sites the twin-line arrows correspond to odd sites. Sing of the probability amplitude is indicated by orientation of the arrow, i.e., plus (minus) corresponds to directed up (down) arrow. The configurations shown in Fig.1 (a) and (b) are characterized by same energy, hence we can except the existence of the solution of the equations (3) which describe the domain wall separated these two configurations. However, there we will not consider this case.

The insertion of the ansatz $\tilde{a}_{2j} = (-1)^j a(\tau)$ and $\tilde{b}_{2j+1} = (-1)^j b(\tau)$ into equations (3) results in following system of equations for the probability amplitudes

$$i\frac{\partial a}{\partial\tau} = \beta_a|a|^2a, \quad i\frac{\partial b}{\partial\tau} = \beta_b|b|^2b \quad (4)$$

Equations (4) show that populations of the sites of each sublattice are indepen-

dent. It is convenient rewrite the equations (4) in term of real variables

$$a(\tau) = u(\tau) \exp\{\varphi_a(\tau)\}, \quad b(\tau) = w(\tau) \exp\{\varphi_b(\tau)\}$$

that leads to the system of simple real equations

$$\frac{\partial u}{\partial \tau} = 0, \quad \frac{\partial w}{\partial \tau} = 0, \quad \frac{\partial \varphi_a}{\partial \tau} = -\beta_a u^2, \quad \frac{\partial \varphi_b}{\partial \tau} = -\beta_b w^2.$$

Solutions of these equations read as

$$u(\tau) = u_0, \quad w(\tau) = w_0, \quad \varphi_a(\tau) = \varphi_{a0} - \beta_a u_0^2 \tau, \quad \varphi_b(\tau) = \varphi_{b0} - \beta_b w_0^2 \tau \quad (5)$$

Choosing of the initial phases we can state the configuration of the initial probability amplitude distribution as it shown in Fig.1 (a) ($\varphi_{a0} = \varphi_{b0} = 0$), or in Fig.1(b) ($\varphi_{a0} = 0, \varphi_{b0} = \pi$).

4 Stability analysis for stationary distribution

Stability of the solution found above will be analyzed in the framework of the linear stability theory. Let us consider the small perturbations of the stationary distribution

$$\tilde{a}_{2j} = (-1)^j a(\tau) + \delta a_{2j}, \quad \tilde{b}_{2j+1} = (-1)^j b(\tau) + \delta b_{2j+1}, \quad (6)$$

with

$$a(\tau) = u_0 \exp\{-\beta_a u_0^2 \tau\}, \quad b(\tau) = w_0 \exp\{-\beta_b w_0^2 \tau\}$$

The initial phases are chosen in the following form: $\varphi_{a0} = 0, \varphi_{b0} = 0$.

The linear equations associated with (3) read as

$$i \frac{\partial}{\partial \tau} \delta a_{2j} = -(\delta b_{2j-1} + \delta b_{2j+1}) + 2\beta_a u_0^2 \delta a_{2j} + \beta_a a^2 \delta a_{2j}^*, \quad (7)$$

$$i \frac{\partial}{\partial \tau} \delta b_{2j+1} = -(\delta a_{2j} + \delta a_{2j+2}) + 2\beta_b w_0^2 \delta b_{2j+1} + \beta_b b^2 \delta b_{2j+1}^*, \quad (8)$$

If one substitute $\delta a_{2j}(\tau) = p_{2j}(\tau) \exp\{-\beta_a u_0^2 \tau\}$, $\delta b_{2j+1}(\tau) = q_{2j+1}(\tau) \exp\{-\beta_b w_0^2 \tau\}$, than (7) and (8) can be rewritten as

$$i \frac{\partial}{\partial \tau} p_{2j} = -(q_{2j-1} + q_{2j+1}) \exp\{i(\varphi_b - \varphi_a)\} + \beta_a u_0^2 (p_{2j} + p_{2j}^*),$$

$$i \frac{\partial}{\partial \tau} q_{2j+1} = -(p_{2j} + p_{2j+2}) \exp\{i(\varphi_a - \varphi_b)\} + \beta_b w_0^2 (q_{2j+1} + q_{2j+1}^*).$$

Assume that the constant probability amplitudes (or the site population of superlattice) are related by the following expression

$$\beta_a u_0^2 = \beta_b w_0^2 = \lambda_1. \quad (9)$$

In this case the phase difference $\varphi_b - \varphi_a$ will be constant. We can put it to zero.

Thus, the system of linear equations for small perturbations takes the form

$$\begin{aligned} i \partial p_{2j} / \partial \tau &= -(q_{2j-1} + q_{2j+1}) + \lambda_1 (p_{2j} + p_{2j}^*), \\ i \partial q_{2j+1} / \partial \tau &= -(p_{2j} + p_{2j+2}) + \lambda_1 (q_{2j+1} + q_{2j+1}^*). \end{aligned} \quad (10)$$

Substitution of the following expressions

$$\begin{aligned} p_{2j} &= A \exp(2ijkl) + B \exp(-2ijkl) \\ q_{2j+1} &= C \exp\{i(2j+1)kl\} + D \exp\{-i(2j+1)kl\} \end{aligned} \quad (11)$$

into the differential-difference equations (10) leads to the system of linear differential equations

$$\begin{aligned} i \partial A / \partial \tau &= -2 \cos kl C + \lambda_1 (A + B^*), & i \partial A^* / \partial \tau &= 2 \cos kl C^* - \lambda_1 (A^* + B), \\ i \partial B / \partial \tau &= -2 \cos kl D + \lambda_1 (A^* + B), & i \partial B^* / \partial \tau &= 2 \cos kl D^* - \lambda_1 (A + B^*), \\ i \partial C / \partial \tau &= -2 \cos kl A + \lambda_1 (D^* + C), & i \partial C^* / \partial \tau &= 2 \cos kl A^* - \lambda_1 (C^* + D), \\ i \partial D / \partial \tau &= -2 \cos kl B + \lambda_1 (C^* + D), & i \partial D^* / \partial \tau &= 2 \cos kl B^* - \lambda_1 (D^* + C). \end{aligned}$$

It is convenient introduce new variable $\xi = \lambda_1 \tau$ and constant parameter $\mu = 2 \cos kl / \lambda_1 = 2 \cos kl / \beta_a u_0^2$. From the foregoing equations one can obtain the system of equations of second order

$$\begin{aligned} \partial^2 A / \partial \xi^2 &= -\mu^2 A + 2\mu C, & \partial^2 C / \partial \xi^2 &= -\mu^2 C + 2\mu A, \\ \partial^2 B / \partial \xi^2 &= -\mu^2 B + 2\mu D, & \partial^2 D / \partial \xi^2 &= -\mu^2 D + 2\mu B. \end{aligned} \quad (12)$$

Now, the characteristic equation for this system of equations (12) can be determined easily

$$\Upsilon(\sigma) = \text{Det} \begin{pmatrix} \sigma^2 - \mu^2 & 2\mu & 0 & 0 \\ 2\mu & \sigma^2 - \mu^2 & 0 & 0 \\ 0 & 0 & \sigma^2 - \mu^2 & 2\mu \\ 0 & 0 & 2\mu & \sigma^2 - \mu^2 \end{pmatrix} = 0 \quad (13)$$

Stability of the solutions of the equations (5) is determined by the roots of this equation, which can be written as

$$\sigma_{\pm}^2 = \mu^2 \pm 2|\mu| = (|\mu| \pm 1)^2 - 1.$$

Instability of the configuration of site population under consideration means that imaginary part of the any root is not zero. But if σ_{\pm}^2 is positive one, then $\text{Im}\sigma = 0$. One should note that $\sigma_+^2 \geq 0$ for any $|\mu|$, whereas $\sigma_-^2 \geq 0$ only at $|\mu| \geq 2$. Hence, one can conclude that the configuration of site population is stable under following condition

$$|\cos kl| \geq \beta_a u_0^2 \quad (14)$$

In terms of physical meaning variable this inequality is read as

$$U_{bb}w_0^2 = U_{aa}u_0^2 \leq |J_0||\cos kl|. \quad (15)$$

Else, the modulation instability takes place if

$$|\cos kl| < \beta_a u_0^2 \quad (16)$$

The amplitude of small perturbations varies as $\exp(i\sigma_{\pm}\xi) = \exp(i\sigma_{\pm}\lambda_1\tau)$. As it was indicated above exponential growing of the amplitude is related with parameter $\sigma_- \lambda_1$. The imaginary part of $\sigma_- \lambda_1$ is the instability increment $G(k)$, i.e., :

$$G^2(k) = 4|\cos kl| (\beta_a u_0^2 - |\cos kl|) \quad (17)$$

If we consider the first Brillouin zone $-\pi/2 \leq kl \leq \pi/2$, then the stability region lies into interval $-\arccos(\beta_a u_0^2) < kl < \arccos(\beta_a u_0^2)$. The instability regions are determined by the inequalities $-\pi/2 < kl < -\arccos(\beta_a u_0^2)$, $\arccos(\beta_a u_0^2) < kl < \pi/2$. One can found that the instability increment is zero at boundary points of these regions. Maximum of the increment placed at points $k_m l = \pm \arccos(\beta_a u_0^2/2)$ and maximum magnitude of increment is equal to $G_m = G(k_m) = \beta_a u_0^2$. And at $\beta_a u_0^2 > 2$ the increment, ones taken at point $k_m l = 0$, has maximum value $G_m = 2\sqrt{\beta_a u_0^2 - 1}$.

Fig.2 show the dependence of the instability increment on wave number of the weak perturbation and on nonlinearity parameter $\lambda_1 = \beta_a u_0^2 = \beta_b w_0^2$. This parameter is defined by the population of the superlattice sites. As one can see increasing of the λ_1 results in decreasing of the stability region. At $\lambda_1 \geq 1$ the instability region occupies first Brillouin zone totally. All solutions (5) are instable.

5 Numerical analysis

The aim of numerical simulation is study of the evolution of stationary configuration of the site population found above in response to a weak harmonic perturbation. Throughout this simulation relation $\beta_a u_0^2 = \beta_b w_0^2$ is assumed. The equations (3) with $\delta = 0$ were solved at the following initial ($\tau = 0$) conditions

$$\begin{aligned}\tilde{a}_{2j}(0) &= (-1)^j u_0 \exp\{-i\lambda_1 \tau\} + \delta a \cos(kl(2j)) \\ \tilde{b}_{2j+1}(0) &= (-1)^j w_0 \exp\{-i\lambda_1 \tau\} + \delta a \cos(kl(2j+1))\end{aligned}$$

There we consider the periodic boundary condition: $b_{n+1} = b_1$, $a_{n+2} = a_2$, where n is total number of sites in superlattice. The superlattice length was chosen to be multiple of the half-period of perturbation.

As example we represent results of the numerical simulation of the modulational instability in superlattice containing 400 sites, where the initial values for probability amplitudes are $a(0) = 2$, $b(0) = 3$, perturbation amplitude is $\delta a = 0.01$ and perturbation wave number is $k = 0.039/l$ (l is distance between neighbor sites). If nonlinearity parameter $\lambda_1 = \beta_a u_0^2$ is over one the modulational instability manifests itself causing an exponential growth of small perturbations of the harmonic wave (Fig.3) and (Fig.4). By using the initial value probability amplitude of population for site of a -type, (i.e., $a(0)$) and the same value at time τ , i.e. $a(\tau)$, one can calculate the instability increment according to formula $G_{num} = \ln[(a(\tau) - a(0))/\delta a]/\tau$ (the same we can done for site of b -type). On the other hand, value of the increment G is determined by formula (17). For $\beta_a u_0^2 = 2.5$ (that corresponds to $\beta_a = 0.625$ and $\beta_b = 0.277$) we

obtained $G_{num} = 2$ and $G = 2.4$. If the value $\beta_a u_0^2$ is described up to one under condition that all parameters of system are fixed, instability persists. However, the instability increment reduces progressively downstream. For $\beta_a u_0^2 < 1$ the regions of stability and instability appeared. Thus we can conclude that instability state of the site population is typical for high populations of the sites in superlattice and for the case, where an on-site interaction between bosons dominates over photo-induced transport. Otherwise one can observe stable state picture: oscillation of an excess population near stable value, as shown in Fig.5.

Now we consider dynamics of the perturbations with different spatial frequencies. The nonlinearity parameter $\beta_a u_0^2$, which put to be less than one, is fixed, but wave number of the harmonic perturbation k (see expression for initial conditions) will be varied. We can expect transition from stability to instability at $kl = \arccos(\beta_a u_0^2)$ because of the stability region lies into interval $-\arccos(\beta_a u_0^2) < kl < \arccos(\beta_a u_0^2)$. Put $\beta_a u_0^2 = 0.99$ ($\beta_a = 0.2475$, $\beta_b = 0.11$). Fig.6 and Fig.7 represent the time dependences of the site populations (i.e., square of modulus of probability amplitudes) respectively for $k_1 l = 0.10244$ ($\cos(k_1 l) = 0.9948$) and $k_1 l = 0.18124$ ($\cos(k_1 l) = 0.9836$).

6 Conclusion

In this work we have studied Bose-Einstein condensates in the one dimensional optical superlattice with two kinds of microscopic potentials (sites of the lattice). It was assumed that the deep of these potentials is enough the system to be described by the Bose-Hubbard model. Steady state distribution of the probability amplitudes of site population was found on base of the earlier derived system of equations (3) [22], determining the dynamic of probability amplitude of sites filling (or the probability to find the boson in this site) in superlattice. Feature of this stationary state is the phase alternating, i.e., phases of probability amplitude of one-type sites change on π while going from one site to another. The stability of such state of BEC for small perturbations depends on the nonlinearity parameter $\beta_a u_0^2$ and also on the frequency of modulation by

itself. When the value of $\beta_a u_0^2$ exceeds one, the found distribution is unstable for all wave numbers from Brillouin zone. However, when $\beta_a u_0^2$ is less than one the instability region in the Brillouin zone are determined by inequalities $-\pi/2 < kl < -\arccos(\beta_a u_0^2)$ and $\arccos(\beta_a u_0^2) < kl < \pi/2$. So, the bosons distribution is modulationally unstable for short wavelength perturbation, whereas the long wavelength perturbations dump out. If the nonlinearity parameter (or the average number of bosons in one site of superlattice) decreases, the instability region reduces. The values of the instability increment found by analytically were compared with the results of numerical simulation, and good agreement between them was found in the field, where the linear analysis of stability is valid.

The nonlinear regime of the modulational instability was studied by using the numerical simulation. It was shown that the number of maxima in population distribution on sites appears. It should be remarked that the preliminary calculations demonstrate the development of chaotic behavior of the considered system.

It is necessary to notice that in ordinary optical lattice the equations of motion for the probability amplitudes of the sites population in continual limit may be transformed into the nonlinear Schrödinger equation having soliton solutions. It describes approximately the spatial solitons in ordinary optical lattices. In case under considering here the equations of motions (3) in continual limit result in more complex equations, which are not like to be completely integrable. So it is not necessary to expect formation of solitons chain as result of modulational instability. Inelastic interaction between solitary spatial waves appearing there, likely will leads to chaotic bosons distribution per sites.

Finally, we assume it not required an especially effort for generalisation of the present results on the 2D cases, e.g., for the simple square or cubic superlattice.

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References

- [1] K. Berg-Sørensen, K. Mølmer, Phys.Rev. A **58**, 1480 (1998)
- [2] A. Kastberg, W.D. Phillips, S. L. Rolston, R. J. Spreeuw, P. S. Jessen, Phys. Rev. Lett. **74**, 1542 (1995).
- [3] L. Guidoni, P. Verkerk, Phys. Rev. A **57**, R1501 (1998).
- [4] M.P.A. Fisher , P. B. Weichman, G. Grinstein, D. S.Fisher, Phys.Rev. B. **40**, 546 (1989).
- [5] A. Hemmerich, M. Weidemuller, T.Esslinger, C. Zimmermann, T. Hensch, Phys.Rev.Lett. **75**, 37 (1995).
- [6] O. Morsch, M. Oberthaler, Rev.Mod.Phys. **78**, 179 (2006).
- [7] Dae-Il Choi, Qian Niu, Phys. Rev. Lett. **82**, 2022 (1999).
- [8] F.Kh. Abdullaev, B.B. Baizakov, S.A. Darmanyany, et al., Phys. Rev. A **64**, 043606 (2001).
- [9] L.P. Pitaevskii, Phys. Usp. **41**, 569 (1998).
- [10] F. Dalfovo, S. Giorgin, L.P. Pitaevskii, Rev. Mod. Phys. **71**, 463 (1999).
- [11] A.J. Leggett, Rev. Mod. Phys. **73**, 307 (2001)
- [12] D. Jaksch, C. Bruder, J.I. Cirac, C.W. Gardiner, P. Zoller, Phys.Rev.Lett. **81**, 3108 (1998).
- [13] F. Massel, V. Penna, Phys.Rev. A. **72**, 053619 (2005).
- [14] O. Fialko, Ch. Moseley, K. Ziegler, Phys.Rev. A **75**, 053616 (2007)

- [15] D. Jaksch, P. Zoller, *Ann.Phys.* **315**, 52 (2005).
- [16] A. Goerlitz, T. Kinoshita, T.W. Haensch, A. Hemmerich, *Phys.Rev. A.* **64**, 011401(R) (2001).
- [17] P. Buonsante, V. Penna, A. Vezzani, *Phys.Rev. A.* **70**, 061603(R) (2004).
- [18] Chou-Chun Huang, Wen-Chin Wu, *Phys.Rev. A.* **72**, 065601 (2005).
- [19] A.B. Bhattacharjee, *J.Phys. B.* **40**, 143 (2007).
- [20] D. Witthaut, E.M. Graefe, S. Wimberger, H.J.Korsch, *Phys.Rev. A.* **75**, 013617 (2007).
- [21] A.I. Maimistov, S.O. Elyutin, *Izv.RAS, ser.phys.* **68**, 264 (2004).
- [22] A.I. Maimistov, *Optics and spectroscopy* **97**, 920 (2004).
- [23] B.B.Baizakov, V.V.Konotop, M. Salerno, *J.Phys. B.* **35**, 5105 (2002).
- [24] V.V.Konotop, M. Salerno, *Phys.Rev. A.* **65**, 021602(R) (2002).
- [25] Guang-Ri Jin, Chul Koo Kim, Kyun Nahm, *Phys.Rev. A.* **72**, 045601 (2005).

FIGURE CAPTIONS

Fig. 1. Two allowed configurations of the probability amplitudes distribution in superlattice (a) $\varphi_{a0} = \varphi_{b0} = 0$, (b) $\varphi_{a0} = 0, \varphi_{b0} = \pi$.

Fig. 2. Instability increment $G(k)$ versus k and λ_1 .

Fig. 3. Strong modulational instability. Evolution of the site populations in superlattice for nonlinearity parameter $\beta_a u_0^2 = 2.5$.

Fig. 4. Evolution of the sites populations for nonlinearity parameter $\beta_a u_0^2 = 2.5$. Solid line corresponds to site with maximum of value of initial perturbation, dash line corresponds to site with maximum of negative value of initial perturbation, dot lines correspond to sites with intermediate value of the initial perturbations.

Fig. 5. Same as in Fig.4 but for nonlinearity parameter $\beta_a u_0^2 = 0.8$. and the dot lines correspond to sites with minimum value of initial perturbation.

Fig. 6. Dynamic of weak harmonic perturbation with $k_1 l = 0.10244$, of the stability region..

Fig. 7. Same as in previous figure, but for harmonic perturbation with $k_1 l = 0.18124$. One can see, that the distribution is instable now ($G_{num} = 0.08$, $G = 0.14$).

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